Locking and unlocking of the counterflow transport in $\nu = 1$ quantum Hall bilayers by tilting of magnetic field

D. V. Fil[1,2]

[1]*Institute for Single Crystals, National Academy of Sciences of Ukraine,*

Lenin Avenue 60, Kharkov 61001, Ukraine

[2]*Max Planck Institute for Solid State Research,*

Heisenbergstrasse 1, D-70569 Stuttgart, Germany

Abstract

The counterflow transport in quantum Hall bilayers provided by superfluid excitons is locked at small input currents due to a complete leakage caused by the interlayer tunneling. We show that the counterflow critical current I_c^{CF} above which the system unlocks for the counterflow transport can be controlled by a tilt of magnetic field in the plane perpendicular to the current direction. The effect is asymmetric with respect to the tilting angle. The unlocking is accompanied by switching of the systems from the d.c. to the a.c. Josephson state. Similar switching takes place for the tunneling set-up when the current flowing through the system exceeds the critical value I_c^{T}. At zero tilt the relation between the tunnel and counterflow critical currents is $I_c^{\mathrm{T}} = 2I_c^{\mathrm{CF}}$. We compare the influence of the in-plane magnetic field component $B_\parallel$ on the critical currents I_c^{CF} and I_c^{T}. The in-plane magnetic field reduces the tunnel critical current and this reduction is symmetric with respect to the tilting angle. It is shown that the difference between I_c^{CF} and I_c^{T} is essential at field $|B_\parallel| \lesssim \phi_0/d\lambda_J$, where ϕ_0 is the flux quantum, d is the interlayer distance, and λ_J is the Josephson length. At larger $B_\parallel$ the critical currents I_c^{CF} and I_c^{T} almost coincide each other.

The idea on exciton superfluidity in electron-hole bilayers[1–3] and especially in quantum Hall bilayers[4–6] with total filling factor $\nu_T = 1$ has obtained a lot of attention in past ten years because of comprehensive experimental study of that problem. In view of possible applications the most important are the counterflow experiments[7–9]. In these experiments the samples with separate assess to the layers are used. Electrical current is injected into one layer in a given end of the Hall bar, is withdrawn from the same layer in the opposite end, and is redirected to the other layer. The currents in the layers have the same value and opposite directions, so they may be provided solely by superfluid magnetoexcitons.

Samples used in the counterflow experiments[7–9] demonstrate a huge increase of conductivity at low temperatures, but they do not demonstrate zero counterflow resistance. We consider that the zero-resistance state can be realized only in quite perfect bilayers. Imperfectness results in emergence of vortices (merons) in the magnetoexciton gas. Meron local concentration is proportional to the deviation of the local filling factor from unity. At rather strong imperfectness merons become uncoupled at all temperatures and their motion perpendicular to the charge transport direction results in a finite counterflow resistance[10–12]. At low degree of imperfectness meron pairs remain bounded and the counterflow resistance should go to zero.

Magnetoexciton superfluidity in bilayers is possible at rather small interlayer separation d (less or of order of the magnetic length ℓ_B). At such a separation the interlayer tunnelling is not negligible and it may influence significantly on the counterflow transport[13–17]. This influence is connected with a formation of another type of vortices - the Josephson ones. The length parameter associated with Josephson vortices is the Josephson length $\lambda_J = \ell_B\sqrt{2\pi\rho_s/t}$, where t is the interlayer tunneling amplitude, and ρ_s is the superfluid stiffness for magnetoexcitons. If λ_J is much smaller than the length of the Hall bar L_x, the effect of locking of the bilayer for the counterflow transport takes place.

The locking occurs at small input current $I_{in} < I_c^{\mathrm{CF}}$ at which a partial Josephson vortex is formed at the source end and the current does not reach the load end. The input critical current is equal to the integral Josephson current for the half of the Josephson vortex: $I_c^{\mathrm{CF}} = 2j_0\lambda_J L_y$, where $j_0 = et/2\pi\hbar\ell_B^2$ is the maximum Josephson current density, and L_y is the width of the Hall bar. One can see that in this state, that is a kind of the d.c. Josephson state[18], the integral Josephson current is proportional to $\sqrt{t}$. At $I_{in} > I_c^{\mathrm{CF}}$ the Josephson vortex chain emerges instead of the partial vortex, and the current reaches the load. Nonzero

current in the load circuit requires nonzero interlayer voltage. This voltage forces the vortex chain to move along the bilayer. Such a state is a kind of the a.c. Josephson state. In this state the leakage is small - the integral average in time Josephson current is proportional to t^2. A finite value of this current is connected with dissipative processes that switch on[13,15] in the a.c. state. The effect of locking and unlocking of the quantum Hall bilayer for the counterflow transport was observed in the recent experiment[19].

In view of possible applications of the exciton superfluidity it is important to control the locking-unlocking effect. In this paper we show that the in-plane component of magnetic field $B_\parallel$ can be used for such a control. We have found that the dependence of $I_c^{\mathrm{CF}}(B_\parallel)$ is asymmetric and one can decrease or increase the critical current by tilting.

We restrict our study with the case of perfect bilayers without free merons and do not consider the influence of the meron induced disorder[20,21] on the critical current.

The switching between the d.c and a.c. Josephson regimes takes place in another experimental set-up called the tunneling one and used for the observation of the Josephson effect in bilayers[22–26]. In this set-up the current is injected into the top layer at one end of the Hall bar and is withdrawn from the bottom layer at the opposite end. In the d.c. Josephson state two partial Josephson vortices are formed at the both ends of the Hall bar and normal co-directed intralayer currents flow in the bulk. At zero in-plane magnetic field the maximum current in the d.c. state is $I_c^{\mathrm{T}} = 2I_c^{\mathrm{CF}}$ (the factor of 2 is due to additive contribution of two ends of the Hall bar). The interlayer voltage in the d.c. state is equal to zero. The transition from the d.c. to the a.c. Josephson regime reveals itself in a sharp drop of the intergal Josephson current. The value of the tunnel critical current can be extracted from the I-V characteristics: the maximum current before its drop is identified as I_c^{T}[25,26]. We would note that in the experiments[25,26] the voltage between two leads in the central part of the sample was much smaller than the voltage between the input and the output leads. It probably means that the bias voltage measured in tunneling experiments is mostly the contact voltage.

In the tunneling set-up the in-plane magnetic field may cause a resonant increase of the integral tunnel current in the a.c. regime[20,27,28] (similar behavior was also observed experimentally[23]). In view of the exact relation between I_c^{T} and I_c^{CF} at zero tilt it is of interest to consider how the tilt changes the critical current I_c^{T}. We find that in balanced bilayers the function $I_c^{\mathrm{T}}(B_\parallel)$ is symmetric and the tilt (irrespective to its sign) results in a

decrease of the tunnel critical current.

We will formulate the problem in terms of the phase of the order parameter φ for the superfluid magnetoexciton gas. The axis x is chosen along the flow direction and the derivative $d\varphi/dx$ determines the intralayer supercurrents

$$j_{s1} = -j_{s2} = \frac{e}{\hbar}\rho_s\left(\frac{d\varphi}{dx} - \frac{eB_yd}{\hbar c}\right). \tag{1}$$

Here and below we imply that $|d\varphi/dx| \ll \ell_B^{-1}$. We specify the case of the phase φ independent of y and the magnetic field tilted in the plane perpendicular to the current direction $(B_\parallel = B_y)$. The Josephson current density reads as

$$j_J = -\frac{e}{\hbar}\frac{t}{2\pi\ell_B^2}\sin\varphi. \tag{2}$$

The quantity j_J is defined as a current that flows from the layer 1 to the layer 2. The intralayer currents contain the uniform counterflow diamagnetic component

$$j_d = -\frac{e^2\rho_sB_yd}{\hbar^2 c}. \tag{3}$$

The diamagnetic effect is rather small: the magnetic susceptibility $\chi = -(e^2/\hbar c)^2\rho_s d/e^2$ is proportional to the square of the fine structure constant. Therefore the difference between the external magnetic field and the field inside the bilayer can be neglected. But the presence of the diamagnetic current is significant for the transport properties.

In the d.c. state the local interlayer voltage is equal to zero $(V_1(\mathbf{r}) = V_2(\mathbf{r}))$ that means the equivalence of electrical fields in the layers $(\mathbf{E_1} = \mathbf{E_2} = \mathbf{E})$. The currents satisfy the stationary continuity equations $dj_{1(2)}/dx \pm j_J = 0$, where the intralayer current is the sum of the supercurrent and the normal current $(j_{1(2)} = j_{s1(2)} + j_{n1(2)})$. Taking into account the condition $j_{s1} = -j_{s2}$, one finds that $\mathbf{j}_{n1} + \mathbf{j}_{n2} = (\hat{\sigma}_1 + \hat{\sigma}_2)\mathbf{E} = const$, where $\hat{\sigma}_i$ is the normal conductivity tensor for the layer i.

In the d.c. state the current in the load circuit should be zero (in the counterflow set-up). Therefore $j_1(L_x) = j_2(L_x) = 0$ that yields $j_{n1}(L_x) + j_{n2}(L_x) = 0$. Thus $j_{n1} + j_{n2} = const = 0$, the electrical field $\mathbf{E} = 0$, and $j_{n1} = j_{n2} = 0$. The continuity equations are reduced to the following equation for the phase

$$\frac{d^2\varphi}{dx^2} = \frac{1}{\lambda_J^2}\sin\varphi. \tag{4}$$

Eq. (4) is the nonlinear pendulum equation in which the time variable is replaced with the space one. Two different types of motion of a nonlinear pendulum (oscillation and rotation)

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correspond to two distinct d.c. Josephson states. They are classified as the vortex-antivortex (VA) chain, and the vortex (V) (or the antivortex (A) chain) state. The word "vortex" ("antivortex") stands for the Josephson vortices with the positive (negative) vorticity.

The currents in the VA state have the form

$$j_{s1}(x) = j_d + j_c\sqrt{\eta}\,\mathrm{cn}\left(\frac{x-x_0}{\lambda_J},\eta\right),$$

$$j_J(x) = \frac{j_c\sqrt{\eta}}{\lambda_J}\,\mathrm{dn}\left(\frac{x-x_0}{\lambda_J},\eta\right)\mathrm{sn}\left(\frac{x-x_0}{\lambda_J},\eta\right).$$

(5)

The V (A) state configuration of currents is described by the equation

$$j_{s1}(x) = j_d \pm \frac{j_c}{\sqrt{\eta}}\mathrm{dn}\left(\frac{x-x_0}{\lambda_J\sqrt{\eta}},\eta\right),$$

$$j_J(x) = \pm\frac{j_c}{\lambda_J}\mathrm{sn}\left(\frac{x-x_0}{\lambda_J\sqrt{\eta}},\eta\right)\mathrm{cn}\left(\frac{x-x_0}{\lambda_J\sqrt{\eta}},\eta\right).$$

(6)

In Eqs. (5), (6) $j_c = 2e\rho_s/\lambda_J\hbar$, is the critical current density, and $\mathrm{sn}(x,\eta)$, $\mathrm{cn}(x,\eta)$ and $\mathrm{dn}(x,\eta)$ are the Jacobi elliptic functions. The parameter η is in the range $(0,1]$. This parameter is connected with the period of the vortex chain. At $\eta \to 1$ the period goes to infinity, and Eq. (5), as well as Eq. (6), describes a single vortex centered at x_0.

The energy of the Josephson vortex state is given by the equation

$$E = \int d^2r \left[\frac{1}{2}\rho_s\left(\frac{d\varphi}{dx} - \frac{eB_yd}{\hbar c}\right)^2 - \frac{t}{2\pi\ell_B^2}\cos\varphi\right].$$

(7)

The conditional minimum of the energy (7) at given boundary conditions for the input and output currents determines the vortex configuration.

Prior to consider the critical current problem we would remind that Josephson vortices can emerge at zero input current, as well[29,30]. If

$$|B_y| > B_c = \frac{4\phi_0}{\pi^2 d\lambda_J}$$

($\phi_0 = hc/2e$ is the flux quantum) Josephson vortices penetrate into the bulk of an isolated bilayer $(j_{1(2)}(0) = j_{1(2)}(L_x) = 0)$ and a vortex chain structure with the period of order of λ_J is formed. The in-plane critical field B_c is analogous to the the critical field H_{c1} for a long Josephson contact between two superconductors. At $|B_y| \leq B_c$ a state with only two partial vortices situated at the opposite ends of the Hall bar is realized. These partial vortices joint counterflow diamagnetic intralayer currents into the circular diamagnetic current.

For the counterflow set-up the critical current density $j_c^{\mathrm{CF}}(B_y)$ can be found as follows. In the d.c. state the normal current is equal to zero. Thus $j_{s1}(0) = j_{in}$, where j_{in} is the input current density, and $j_{s1}(L_x) = 0$. We imply that the phase $\varphi(x)$ is a continuous function of x. It corresponds to the vortex state with the same η in the whole system. For a given state the intralayer current varies in a certain range determined by the parameter η and the in-plane field B_y. It is the range $[j_d - j_c\sqrt{\eta}, j_d + j_c\sqrt{\eta}]$ for the VA state, the range $[j_d + j_c\sqrt{(1-\eta)/\eta}, j_d + j_c\sqrt{1/\eta}]$ for the V state, and the range $[j_d - j_c\sqrt{1/\eta}, j_d - j_c\sqrt{(1-\eta)/\eta}]$ for the A state. The counterflow d.c. state can be realized if at least one of the ranges enumerated above contains both j_{in} and zero. The situation is symmetric with respect to the change of sign of both j_{in} and B_y, and it is enough to consider only positive j_{in}.

For the further analysis it is convenient to define the characteristic in-plane magnetic field

$$B_c' = \frac{2}{\pi} \frac{\phi_0}{d\lambda_J} \qquad (8)$$

determined by the condition $|j_d| = j_c$. It is larger than B_c ($B_c' = \pi B_c/2$).

Let us first consider the case of positive B_y ($j_d < 0$). The VA state may satisfy the boundary conditions if $j_{in} \leq j_c - |j_d|$. The V state is possible if for the same η two inequalities $j_{in} < j_c\sqrt{1/\eta} - |j_d|$ and $j_c\sqrt{(1-\eta)/\eta} - |j_d| < 0$ are fulfilled. That yields the following restriction on the input current: $j_{in} \leq \sqrt{j_c^2 + j_d^2} - |j_d|$.

At $-B_c' < B_y < 0$ ($j_d > 0$) the d.c. state is possible up to $j_{in} = j_d + j_c$. Indeed, the VA state satisfies the boundary conditions at $j_d < j_{in} \leq j_d + j_c$, and the A state - at $j_{in} \leq j_d$. At $B_y > -B_c'$ the VA state is not possible, and the A state may satisfy the d.c. boundary conditions only for the input current $j_{in} < j_d - \sqrt{j_d^2 - j_c^2}$.

Thus the dependence of the counterflow critical current density on B_y has the form

$$j_c^{\mathrm{CF}}(B_y) = j_c \begin{cases} -\frac{B_y}{B_c'} - \sqrt{\left(\frac{B_y}{B_c'}\right)^2 - 1} & \text{at } B_y < -B_c' \\ 1 - \frac{B_y}{B_c'} & \text{at } -B_c' \leq B_y < 0 \\ \left[\sqrt{1 + \left(\frac{B_y}{B_c'}\right)^2} - \frac{B_y}{B_c'}\right] & \text{at } B_y > 0 \end{cases} \qquad (9)$$

The dependence (9) is shown in Fig. 1. One can see that at $|B_y| \lesssim B_c'$ this dependence is essentially asymmetric one. Such an asymmetry is connected with that the counterflow current and the diamagnetic current can be co-directed or oppositely directed depending on the sign of the tilting angle. The tilting angle that corresponds to $B_y = B_c'$ is rather small: $\phi_{\mathrm{tilt}}^c \approx 2\ell_B^2/(d\lambda_J)$.
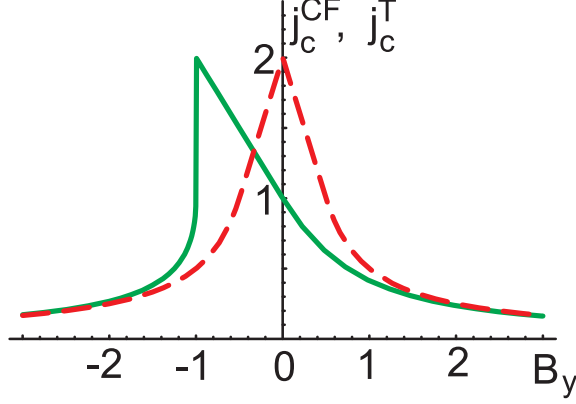
FIG. 1: Critical current densities (in j_c units) vs. the in-plane magnetic field (in B'_c units). The counterflow critical current j_c^{CF} is shown by solid line, the tunnel critical current j_c^{T} - by dashed line.

Let us say some words on the role of critical field B_c. The dependence $j_c^{\mathrm{CF}}(B_y)$ is continuous one at $|B_y| = B_c$. But the vortex structure that corresponds to the energy minimum changes significantly at this point. At $|B_y| < B_c$ the V and A states are the states with only two partial vortices (antivortices) at the opposite ends. But if $|B_y|$ exceeds B_c these states are transformed into multivortex ones. The VA state with minimal energy is the state with only a partial vortex at one end and a partial antivortex at the opposite end irrespective of the value of B_y.

Let us now switch to the tunneling set-up. In this set-up $j_1(0) = j_2(L_x) = j_{in}$ and $j_2(0) = j_1(L_x) = 0$. Since the counterflow currents cannot transfer the charge between two ends, normal currents are nonzero and their sum is equal to the input current $j_{n1} + j_{n2} = j_{in} = const$. The difference $j_{n1} - j_{n2} = const$, as well. Thus the normal current does not enter into the continuity equation, and the latter is reduced to the equation for the phase (4).

Here we specify the case of balanced bilayers in which $j_{n1} = j_{n2}$, and the supercurrents satisfy the boundary conditions $j_{s1}(0) = -j_{s1}(L_x) = j_{in}/2$. For given B_y and η we have three ranges of j_{s1} (that coincide with ones given above) for the VA, V and A solutions. In the tunneling set-up the d.c. state can be realized if the quantities $+j_{in}/2$ and $-j_{in}/2$ belong to the same range. For the VA solution the latter condition is fulfilled under the

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following restriction on the value of the input current

$$|j_{in}| < 2(j_c - |j_d|). \tag{10}$$

The V solution may satisfy the boundary condition at negative j_d, and the A solution - at positive j_d. Common for both solutions restriction on j_{in} reads as

$$|j_{in}| < \max[F(\eta)], \tag{11}$$

where the function

$$F(\eta) = 2 \min(|j_d| - j_c\sqrt{\frac{1-\eta}{\eta}}, j_c\frac{1}{\sqrt{\eta}} - |j_d|) \tag{12}$$

contains B_y as a parameter and is defined in the interval $0 < \eta \leq 1$. Let us find η_m that maximizes the function $F(\eta)$. At $|j_d| < j_c/2$ we obtain $\eta_m = 1$ and $F(\eta_m) = 2|j_d|$. At $|j_d| > j_c/2$ the quantity η_m is determined by the equation

$$|j_d| - j_c\sqrt{\frac{1-\eta_m}{\eta_m}} = j_c\frac{1}{\sqrt{\eta_m}} - |j_d| \tag{13}$$

that yields $\sqrt{\eta_m} = 4j_c|j_d|/(4j_d^2 + j_c^2)$ and $F(\eta_m) = j_c^2/2|j_d|$

Comparing the conditions (10) and (11) we obtain the final expression for the tunnel critical current density

$$j_c^T(B_y) = 2j_c \begin{cases} 1 - \frac{|B_y|}{B_c'} & \text{at } |B_y| < B_c'/2 \\ \frac{B_c'}{4|B_y|} & \text{at } |B_y| \geq B_c'/2 \end{cases} \tag{14}$$

The dependence (14) is shown in Fig. 1. One can see that while at $B_y = 0$ the current j_c^T exceeds j_c^{CF} by the factor of two, at $|B_y| \gg B_c'$ these quantities almost coincide each other. The other difference between j_c^T and j_c^{CF} is that the tunneling critical current is symmetric with respect the sign of the tilting angle. The latter property can also be predicted from the symmetry reasons. Note that such a symmetry takes place only in case of balanced bilayers: at nonzero imbalance $j_{n1} \neq j_{n2}$, that results in asymmetric dependence $j_c^T(B_y)$.

In conclusion, we have shown that the locking and unlocking of the quantum Hall bilayer for the counterflow transport can be controlled by tilting of magnetic field. The effect can be observed in the same experimental set-up, where the locking-unlocking effect under variation of the input current was recently discovered[19]. Asymmetric dependence of the critical current on magnetic field is expected in a rather narrow diapason of tilting angles close to zero. We have compared the influence of the in-plane magnetic field on the counterflow critical

current and on the tunnel critical current[25,26]. We find that the difference is essential at small in-plane magnetic fields. The maximum counterflow critical current coincides with the maximum tunnel critical current, but in the first case the maximum is reached at $B_y = -B'_c$, while in the second case - at $B_y = 0$.

It is important to discuss the validity of our results for real experimental systems. The main assumption of our consideration is the existence of a path between the input and the output end that is free from merons and weak links. We imply that the phase of the order parameter is continuous one along this path. Systems, where such a path does not exist, but which have quite long areas without merons may also demonstrate similar behavior. In the latter case the tunnel critical current at $B_y \neq 0$ should be larger than in the case considered in this paper. It is because two ends will work separately.

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[1] S. I. Shevchenko, Fiz. Nizk. Temp. **2**, 505 (1976) [Sov. J. Low Temp. Phys. **2**, 251 (1976)]; S. I. Shevchenko, Phys. Rev. Lett. **72**, 3242 (1994).

[2] Yu. E. Lozovik, and V. I. Yudson, Zh. Eksp. Theor. Fiz. **71**, 738 (1976) [Sov. Phys. JETP **44**, 389 (1976)].

[3] A. V. Balatsky, Y. N. Joglekar, P.B. Littlewood, Phys. Rev. Lett. **93**, 266801 (2004).

[4] H. A. Fertig, Phys. Rev. B **40**, 1087 (1989).

[5] X. G. Wen and A. Zee, Phys. Rev. Lett. **69**, 1811 (1992).

[6] K. Moon, H. Mori, K. Yang, S. M. Girvin, A. H. MacDonald, L. Zheng, D. Yoshioka, and S. C. Zhang, Phys. Rev.B **51**, 5138 (1995).

[7] M. Kellogg, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **93**, 036801 (2004).

[8] R. D. Wiersma, J. G. S. Lok, L. Tiemann, W. Dietsche, K. von Klitzing, D. Schuh, W. Wegscheider, Physica E **35**, 320 (2006).

[9] E. Tutuc, M. Shayegan, and D. A. Huse, Phys. Rev. Lett. **93**, 036802 (2004).

[10] D. A. Huse, Phys. Rev. B **72**, 064514 (2005).

[11] H. A. Fertig, and G. Murthy, Phys. Rev. Lett. **95**, 156802 (2005); B. Roostaei, K. J. Mullen, H. A. Fertig, S. H. Simon, Phys. Rev. Lett. **101**, 046804 (2008).

[12] D. V. Fil, S. I. Shevchenko, Phys. Lett. A **374**, 3335 (2010).

[13] D. V. Fil and S. I. Shevchenko, Fiz. Nizk. Temp. **33**, 1023 (2007) [Low Temp. Phys. **33**, 780 (2007)].

[14] J.-J. Su and A. H. MacDonald, Nature Physics **4**, 799 (2008).

[15] D. V. Fil and S. I. Shevchenko, J. Phys.: Condens. Matter **21**, 215701 (2009).

[16] R. Khomeriki, L. Tkeshelashvili, T. Buishvili, and S. Revishvili, Eur. Phys. J. B **51**, 421 (2006).

[17] J.-J. Su and A. H. MacDonald, Phys. Rev. B **81**, 184523 (2010).

[18] Here the d.c. state is understood as a state in which the current does not contain hf Josephson-type oscillations. It takes place for a d.c. input current, as well as, for a low frequency a.c. input current, if its amplitude is lower than the critical one.

[19] Y. Yoon, L. Tiemann, S. Schmult, W. Dietsche, K. von Klitzing, W. Wegscheider, Phys. Rev. Lett, **104**, 116802 (2010).

[20] M. M. Fogler and F. Wilczek, Phys. Rev. Lett. **86**, 1833 (2001).

[21] P. R. Eastham, N. R. Cooper, D. K. K. Lee, arXiv1003.5191.

[22] I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **84**, 5808 (2000).

[23] I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **87**, 036803 (2001).

[24] A. R. Champagne, J. P. Eisenstein, L. N. Pfeiffer, and K.W. West, Phys. Rev. Lett. **100**, 096801 (2008).

[25] L. Tiemann, W. Dietsche, M. Hauser, and K. von Klitzing, New J. of Phys. **10**, 045018 (2008).

[26] L. Tiemann, Y. Yoon, W. Dietsche, K. von Klitzing, W. Wegscheider, Phys. Rev. B **80**, 165120 (2009).

[27] A. Stern, S. M. Girvin, A. H. MacDonald, and N. Ma, Phys. Rev. Lett. **86**, 1829 (2001).

[28] L. Balents and L. Radzihovsky, Phys. Rev. Lett. **86**, 1825 (2001).

[29] Z. F. Ezawa, A. Iwazaki, Int. J. Mod. Phys. B **8**, 2111 (1994).

[30] C. B. Hanna, A. H. MacDonald, and S. M. Girvin, Phys. Rev. B **63**, 125305 (2001).